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RESEARCH ARTICLE OPEN ACCESS

A COOPERATIVE GAME THEORY APPLICATION IN CHICKS BROOD FOOD ALLOCATION BY USING SHAPLEY VALUE METHOD IN GOOD YEARS DATA

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ABSTRACT

Game theory as well as cooperative game theory has played a vital role in many fields of research since its introduction within the early twentieth century. During this paper we study food allocation in chick broods from the attitude of cooperative theory of games. We would like to explore whether or not food distribution data fit into the known solution concepts of cooperative game theory to create an economic temperature within the biological field. A primary approach to be handled is that the incontrovertible fact that within the chick brood data we only see the solutions, while the starting position, the game, isn't immediately clear. In and of itself we'd like to reconstruct the game from the solutions given. A second approach is that by using the answer concepts Shapley value we would like to investigate which of those fits best. Most interesting is to specifically address the properties that cause these solutions because these would be most useful to find motivation for the particular solution concept found in nature. The goal is to anticipate moves to create, which is able to cause ultimate victory.

I. INTRODUCTION

Game theory is an autonomous discipline that is used in applied mathematics, social sciences, most remarkably in mathematics, economics, as well as in computer science, biology, engineering, international relations, philosophy, and political science [1]. It was displayed in economics and mathematics to measure economic behaviors including behaviors of firms, markets, and consumers. The cooperative game theory can be contemplated as a modeling procedure that is used to analyze and explain the actions of all players joined in competitive situations and to compare and determine the relative optimality of distinct strategies. The first inspect of games in terms of economics was by Cournot on pricing and production but Neumann (1944) is considered as the founder of the modem game theory [1]. Coined by Shapley (1953), this one-point solution concept introduced in his paper has some desirable properties called efficiency, anonymity, dummy, and additivity [4]. A solution is efficient if it assigns to every game an allocation in such a way that the sum of every marginal contribution of each player will be equal to the value of the grand coalition [1, 2, 3]. Recent works by Forbes (2005, 2007) is employing a financial tool to the study of parental investment in chick, as normally the foremost important investment any organism makes is in its offspring [5]. In cooperative game theory, it can be said that the set of bounded computational capacity of equilibrium payoffs carries only one valuation, that the valuation of the game with penalty approaches the valuation of the one-shot game as the penalty goes to zero [1, 6].

I.1 NOTATIONS

eggs: Shows how many eggs are firstly available in total before the hatching period.
c: In day 1, denotes the number of core eggs (eggs that are hatched in day 1) inside one brood.
m: The number of marginal eggs (eggs that are left/not yet hatched in day 1) in one brood.

brood: Shows how many broods that are available in total.

$\boldsymbol{m_1}$**:** The number of marginal that are hatched on the first day after the core.

$\boldsymbol{m_2}$**:** The number of marginal that are hatched on the second day after the core.

$\boldsymbol{m_3}$**:** The number of marginal that are hatched on the third day after the core.

$\boldsymbol{total}$: In day 8, shows how many chicks that are continue living after one week of feeding.

$\boldsymbol{d8\ val}$: The average number of all chick (both core and marginal) which survive until one week of feeding (i.e. day 8).

$\boldsymbol{m\ av}$: The average of the marginal chick' survival rate which we will use to fill the coalition value for the marginal later in our method.

$\boldsymbol{c\ val}$: The survival rate of each chick.

$\boldsymbol{m_1}$ **val**: The survival rate of starting from the core chick.

$\boldsymbol{m_2}$ **val**: The survival rate of the first marginal chick.

$\boldsymbol{m_3}$ **val**: The survival rate of until the third marginal chick.

$\boldsymbol{v(S)}$: The value of the coalition.

$\boldsymbol{A}$: A matrix with real entries a_{ij}.

$\boldsymbol{x}$: A column vector with real entries x_i.

$\boldsymbol{x^S}$: The payoff vectors.

$\boldsymbol{x^T}$: The transpose of vector **x.**

$\boldsymbol{\{x_1, x_2\}}$: The sequence of vectors x_1, x_2

$\boldsymbol{x^*}$: The optimal value of vector **x**

$\emptyset$: Shapley value

II. STUDY ON CHICK BROOD FAIR ALLOCATION PROBLEM

Recent works by Forbes is employing a financial tool to the study of parental investment in chick, as normally the foremost important investment any organism makes is in its offspring [5]. A key dimension of any investment decision including what proportion to speculate in offspring is a way to balance risk and reward that portfolio theory offers a broad set of analytical tools. An initial complexity for the biologist introduces in a way to translate the economic models into a biological patent. The tool he used is named financial beta and is well-known to the study of parental investment, derived from the capital asset pricing model of recent portfolio theory. Beta provides a measure of the volatility within the price of an asset for a wide market or index. Forbes suggested that the reproductive returns from individual brood structures (e.g., mean edging success in an exceedingly given year) may well be usefully equated to a private asset, which means population reproductive success may well be equated to the market as an entire.

There is another study conducted by Alex Kacelnik, Peter A. Cotton, Liam Stirling, and Jonathan Wright which use the evolutionary theory of games to review Food Allocation among Nestling Starlings, drawing attention on Sibling Competition and also the Scope of Parental Choice. Chick feeding in chick is commonly viewed as a chief example of evolutionary conflict. this can be because the nestlings may benefit by inducing the parent to speculate more within the current brood compared to future ones. additionally, each nestling should benefit by obtaining a greater fraction of the full brood provision than would be optimal for the parent. Current theory suggests that at evolutionary equilibrium, the intensity of signalling (i.e. begging) by the chick should allow the fogeys to spot each chick's needs and to allocate more food to the one that gives the steepest marginal fitness gain per unit of parental resources [5].

III. THE CHICK BROOD DATA IN GOOD YEARS

The chick are puppies who have hatched together on the first day of nesting. Instead, chick hatch for more than a day and are identified as marginal chick. Parental choices for the number of eggs hatching on the first day may be based on their experiences during the early hatching period, or their instincts about the weather and food conditions near the nest. They hit their marginal according to their core every day. Therefore, if you have 2 core chick and 3 margins, the total hatch time is 4 days (1 day for all cores and 3 days for each margin). The raw data in Table 1 below will give you an idea of the number of chickens and chickens that one chick can have, and the number of chick available to chickens and chick. Be found. You can also see how many chicks died in a good week (from day 1 to day 8), so you can see if there are any differences in how parents are assigned.

Table 1: The good year data.

eggs	**Day 1**			**Day 8**					
	c	**m**	**brood**	**c**	**m**	$\boldsymbol{m_1}$	$\boldsymbol{m_2}$	$\boldsymbol{m_3}$	**total**
18	1	0	15	15	0	0	0	0	15
59	1	1	29	27	27	27	0	0	53
120	1	2	40	38	62	33	29	0	95
218	1	3	55	54	124	52	48	24	176
30	2	0	15	28	0	0	0	0	28
236	2	1	78	134	56	56	0	0	186
430	2	2	108	208	140	85	55	0	345
75	2	3	15	29	26	14	9	3	55
81	3	0	27	74	0	0	0	0	74
292	3	1	73	196	33	33	0	0	224
100	3	2	20	55	16	12	3	0	71
32	4	0	8	27	0	0	0	0	27
25	4	1	5	19	2	2	0	0	21

Source: Authors, (2019).

As we've seen within the above data, one brood can contain at maximum 4 core chick, while on the opposite hand, it may also contain at maximum 3 marginal. While the whole number of chick is seven (i.e. there are seven players within the game), we don't have data with four core and three marginal at the identical time. The largest brood we've consisted of 5 chicks, either 2 core with 3 marginals, 3 core with 2 marginals, or 4 cores with 1 marginal. Note that from the story of chick the fogeys will feed the chick that beg louder, which usually are the core chick. Therefore, in a method to calculate the Shapley value, we fancy assuming that the feeding process will always start with the core chick, while the marginals 'fight' over the remaining food after the cores are being fed. This assumption is described later during calculations and experiments of our method. In theory, we also cannot have only marginals without having the core, or having the third and/or the second marginal without having the primary one. But this can be happening in a number of the brood data since there's a break that the egg is missing or being destroyed during the hatching period of the core chick, to not mention the chick that's directly dead after born, leaving only the marginals within the brood. The same case is additionally happening for the marginal. However, later we are going to see that our method excludes this sort of missing data from the calculations, and considers only the feasible coalitions. As a result of being born on different days where the marginals are hatched on a daily basis after the cores, we expect our brood data have a particular property: there exist different weights between core and marginal chick. this can be because the core and also the marginal chick may value their food in numerous ways. We predict that everyone the core chick's c will value their food within the same manner since they're hatched on an identical day (thus could also be as strong as each other) and also the parents consider them to be equally important to continue the family legacy. As a result, the weights difference between the core chick is incredibly small or is ignored. In other words, we assume that competition between the core chick in one brood doesn't exist. However, there exist different weights between the core and also the marginals, in addition to between all the marginals, because the marginal m_i, i=1 to 3, are born consecutively on i days after the core. Note that this weight isn't a body-mass index but a further number that represents how the chick may value their food. To be ready to build a coalition model for the brood food allocation data during the great and also the bad years, we firstly we define an XY-brood game where X and Y denote the quantity of core and marginal children respectively.

As input for the chick brood food allocation model, we use the type of survival rate data A and also the importance weights data I which shows what percentage times a particular style of brood (i.e. core and marginal coalition) appears within the game.

Table 2: Average of the Survival Rate for XY-brood type during the Good Years.

A								I
c	**m**	**c val**	**m av**	m_1 **val**	m_2 **val**	m_3 **val**	**d8 val**	
1	0	1.000	0	0	0	0	1.000	15
1	1	0.931	0.931	0.931	0	0	1.862	29
1	2	0.950	0.838	0.892	0.784	0	2.626	40
1	3	0.982	0.765	0.963	0.889	0.453	3.287	55
2	0	0.933	0	0	0	0	1.867	15
2	1	0.859	0.747	0.747	0	0	2.465	78
2	2	0.963	0.688	0.794	0.514	0	3.234	108
2	3	0.967	0.578	0.933	0.600	0.200	3.667	15
3	0	0.914	0	0	0	0	2.741	27
3	1	0.895	0.465	0.465	0	0	3.150	73
3	2	0.917	4.000	0.632	0.158	0	3.539	20
4	0	0.844	0	0	0	0	3.375	8
4	1	0.950	4.000	4.000	0	0	4.200	5

Source: Authors, (2019).

Having the coalitions S, what value can we choose to be the value of the coalition v(S)? Since we have the average of the survival rate A for each off-springs in every XY-brood, taking into account its importance weight I (i.e. how many times the XY-brood data occur), we can take these values as the value of the coalitions.

Table 3: 21-brood game, Good years.

S	{1}	{2}	{5}	{1,2}	{1,5}	{2,5}	{1,2,5}
$v(S)$	1.000	1.000	0.931	1.867	1.862	1.862	2.465

Source: Authors, (2019).

As we are able to see within the brood data for the good years (Table 2), there are data of 10-brood, 11-brood, 12-brood, 13-brood, 20-brood, 21-brood, 22-brood, 23- brood, 30-brood, 31-brood, 32-brood, 40-brood, and 41-brood. to urge a sense of those data we are having, we are going to calculate the Shapley value ∅ by hand for a few of the smaller brood data, to determine if we are able to get something interesting as a result.

There are two different methods used to calculate the payment vector for the x^S for the coalition S, the key is the standard approach.

Remember that we've at maximum seven players (|N|=7), incorporates a maximum of 4 core players (i=1 to 4) and at maximum 3 marginal players (i=5 to 7).

Thus, we may consider seven different places for every different position of the players.

In this standard approach, the core chick is often placed in anywhere among the four first places while the marginals are placed consecutively within the three last places (in increasing order). Because the core may become the primary, the second, the third, or the fourth player, it is often placed in any of the four first places.

Therefore, we'd like to contemplate the identical survival rates for these four possible places of the core chick in each XY-brood game; Weight is important, but the ideal cover chicken is divided into four possible areas.

Note that whichever chick chooses the primary place is considered because of the first core, and so on. To be clear about this representation, we convert Table 2 of the good year's data into a replacement Table 4, by defining an allocation x^S because the average of each chick's survival rate within the corresponding XY-brood game, taking into consideration the possible coalitions that may be made by all the players involved within the game. For example, if we consider the good years 21-brood game using the quality approach, we are going to have a collection of possible coalitions that consists of a coalition. Since the 2 core chick can choose any of the four first places.

The Table 4 below will show what number players involved in each coalition of a selected XY-brood, what are the possible coalitions exist in a very specific XY- brood, and what are the survival rates of every player involves in those specific coalitions.

Note that the numbers i=1 to 7 in the table denote the players, where i=1 to 4 are core players and i=5 to 7 are marginal players.

Table 4: Survival rates of the players in the existing coalitions (Standard Approach, Good Years).

XY	**Possible Coalitions** S	X^s			
		1 to 4	**5**	**6**	**7**
10	{1},{2},{3},{4}	1.000	0	0	0
11	{1,5},{2,5},{3,5},{4,5}	0.931	0.931	0	0
12	{1,5,6},{2,5,6},{3,5,6},{4,5,6}	0.950	0.892	0.784	0
13	$\{i,5,6,7\},\forall i=1\ to\ 4$	0.982	0.963	0.889	0.453
20	$\{i,j\},\forall i,j=1\ to\ 4, i<j$	0.933	0	0	0
21	$\{i,j,5\},\forall i,j=1\ to\ 4, i<j$	0.859	0.747	0	0
22	$\{i,j,5,6\},\forall i,j=1\ to\ 4, i<j$	0.963	0.794	0.514	0
23	$\{i,j,5,6,7\},\forall i,j=1\ to\ 4, i<j$	0.967	0.933	0.600	0.200
30	$\{i,j,k\},\forall i,j,k=1\ to\ 4, i<j<k$	0.914	0	0	0
31	$\{i,j,k,5\},\forall i,j,k=1\ to\ 4, i<j<k$	0.895	0.465	0	0
32	$\{i,j,k,5,6\},\forall i,j,k=1\ to\ 4, i<j<k$	0.917	0.632	0.158	0
40	{1,2,3,4}	0.844	0	0	0
41	{1,2,3,4,5}	0.950	0.400	0	0

Source: Authors, (2019).

Note that in total we are going to have 54 coalitions if we consider the quality approach. Here we restrict n core players, n=1 to 4, to always be within the first n-places, the first n-places and the remaining 4-n places that seem to have been occupied by the core chick will have a zero value. as an example, if we have a 21-brood where there are two core players and one marginal, then the 2 core players will always fill the primary and also the second places, while the third and also the fourth places remain zero. that's we are going to have only 13 coalitions if we consider this approach. This number is the same because of the number of all existing XY-brood games.

Table 5: Survival rates of the players in the existing coalitions (Restricted Approach, Good Years).

XY	Possible Coalition S	X^s						
		1	2	3	4	5	6	7
10	{1}	1.000	0	0	0	0	0	0
11	{1,5}	0.931	0	0	0	0.931	0	0
12	{1,5,6}	0.950	0	0	0	0.892	0.784	0
13	{1,5,6,7}	0.982	0	0	0	0.963	0.889	0.453
20	{1,2}	0.933	0.933	0	0	0	0	0
21	{1,2,5}	0.859	0.859	0	0	0.747	0	0
22	{1,2,5,6}	0.963	0.963	0	0	0.794	0.514	0
23	{1,2,5,6,7}	0.967	0.967	0	0	0.933	0.600	0.200
30	{1,2,3}	0.914	0.914	0.914	0	0	0	0
31	{1,2,3,5}	0.895	0.895	0.895	0	0.465	0	0
32	{1,2,3,5,6}	0.917	0.917	0.917	0	0.632	0.158	0
40	{1,2,3,4}	0.844	0.844	0.844	0.844	0	0	0
41	{1,2,3,4,5}	0.950	0.950	0.950	0.950	0.400	0	0

Source: Authors, (2019).

We will discuss two methods of calculation the Shapley value ∅ for 12-brood by hand. the first one is finished for every possible ordering of the grand coalition, while the second concerns the elimination of orders that are considered unnecessary before starting calculating the value. to grasp how this calculation method works, we are visiting denote a group of possible orders P as any possible orders of the chick once they're being fed by the parents: starting from the firstly fed chick, until every chick within the corresponding XY-brood data is being fed. as an example, order 1-5-6 within the 12- brood game means the core chick is being fed within the primary place, followed by the first and also the second marginals consecutively. When chick parents come to the nest bringing the foods for his or her chick, whichever chick that begs harder are fed first with usually the most important amount of food, and vice versa; chick that's being fed last will get just the rest. As food given from parents is perhaps the sole source of the chick' nutrition's, a minimum of until they're ready to y and appearance for an additional source of food, this food is extremely important for them to survive. Therefore, we may logically assume that the chick's order of being fed will affect their survival rate. Using this assumption, it's possible to calculate the Shapley value by taking the common of the chick' survival rate within the corresponding brood data to be interpreted because the amount of food the chicks have gotten from their parents which can help them to survive. a mean of 1.000 for a chick's survival rate might be translated as: the chick is getting 100% of food that it must survive. so as to calculate the Shapley value by hand using the interpretation above, we define the subsequent allocation procedure:

1. Consider the XY-brood game during either the good or the bad year's period under the restricted approach. Make a coalition table for the XY- brood. As an example, now we consider the good years 12-brood game, looking at Table 2 for the chick' average survival rate data during the good years, notice that we use the total sum of all chick' survival rate in 12-brood to fill in the value of the grand coalition, while the marginals average m av of the 11-brood and 12-brood are used to fill in the value of the coalition {5} and {6}, respectively. To fill in the value for coalition {1,5} and {1,6} we use the sum of the survival rate for core chick in 11-brood with the m av of 11-brood and 12-brood respectively. Finally, the sum of m_1and m_2 survival rate of the 12-brood is used to fill in the value for coalition {5,6}. Thus, we have a coalition table for the 12-brood game as follows:

Table 6: 12-brood game, good years.

S	{1}	{2}	{5}	{1,2}	{1,5}	{2,5}	{1,2,5}
$v(S)$	1.000	0.931	0.838	1.862	1.769	1.676	2.626

Source: Authors, (2019).

2. List every possible orders of the grand coalition that correspond to this XY-brood data.

3. In order to be able to fill within the 'right' value that each player will get in keeping with their possible ordering, we adapt the identical Shapley procedure. This way, we are going to divide the worth of the grand coalition 'fairly' by considering the orders and also the value that are 'claimed' by each coalition. As an example, from Table 5, we all know that player 1 in coalition {1} is 'claiming' a median of player 1 for its survival rate, while player 5 and 6 in coalition {5} and {6} and are 'claiming' a median of 0.931 and 0.838, respectively. If we take into consideration order 1-5-6 of the players within the grand coalition, we are going to firstly allocate 1.000 for player 1; precisely the same amount as what it claims. to come to a decision what proportion should player 5 gets, we glance at Table 6 and see that 1.862 is that the value of coalition {1,5}. Since we already give player 1 a worth of 1.000, the remaining value of 0.862 are going to be the quantity which is given to player 5. confine mind that the sum of each player's value has to be capable the worth of the grand coalition. Since a complete of 1.862 has already been given to player 1 and 5, player 6 will get the rest of the grand coalition value; which is 0.764. Doing the identical procedures to each possible orders, we are going to get a Shapley value calculation table as shown below. Note that notation ∅ denotes the Shapley value of every player involved within the grand coalition.

Table 7: Shapely value of 12-brood game, good years.

Possible Orders P	Player			
	1	5	6	Total
1-5-6	1.000	0.862	0.764	
1-6-5	1.000	0.857	0.769	
5-1-6	0.931	0.931	0.764	
5-6-1	0.950	0.931	0.745	
6-1-5	0.931	0.857	0.838	
6-5-1	0.950	0.838	0.838	
∅	0.960	0.879	0.786	2.626

Source: Authors, (2019).

4. Now we compare the values we got from observing the common survival rates of every chick within the corresponding XY-brood data, which we denote as Observ. to the Shapley values we got from calculation. so as to induce these observation values, we want to appear at Table 5 (restricted approach) and find the common survival rate of every chick within the corresponding XY-brood. as an example, the observation values for player 1, player 5, and player 6 in 12-brood game in line with Table 5 are 0.950, 0.892, and 0.784, respectively. For easier comparison, we are going to add these observation values into the Shapley value calculation table we made within the previous step, resulting this table below:

Table 8: Possible orders for 12-brood game, good years.

Possible Orders P	Player			
	1	**5**	**6**	**Total**
1-5-6	1.000	0.862	0.764	
1-6-5	1.000	0.857	0.769	
5-1-6	0.931	0.931	0.764	
5-6-1	0.950	0.931	0.745	
6-1-5	0.931	0.857	0.838	
6-5-1	0.950	0.838	0.838	
∅	0.960	0.879	0.786	2.626
Observ.	0.950	0.892	0.784	2.626

Source: Authors, (2019).

5. Since there's almost no case within the chick broods where the marginal chicks are being fed before the core chick, now we fancy to jump over the unfeasible orders from the Shapley value calculation table and consider only the cases when the core chick are being fed before the marginals. We now have a brand new Shapley value calculation table with a collection of feasible orders F rather than possible ones. Below is that the new Shapley value calculation table for the 12-brood game:

Table 9: Feasible orders for 12-brood game, good years.

Feasible Orders P	Player			
	1	**5**	**6**	**Total**
1-5-6	1.000	0.862	0.764	
1-6-5	1.000	0.857	0.769	
∅	1.000	0.8595	0.7665	2.626
Observ.	0.950	0.892	0.784	2.626

Source: Authors, (2019).

Again, we compare the Shapley value ∅ with the observation value to see if parental favoritism exists within the case of specific XY-brood data. Notice that within the case of 12-brood, the Shapley values ∅ for the core chick that we dawned on both cases are larger than the observation values. Thus there is no tendency of parents favoriting the core chick in line with this Shapley value solution in 12-brood. On the alternative hand, the Shapley value for the marginal is almost always larger within the observations rather than within the calculation; aside from the second marginal within the case of taking all possible orders P into the calculation. Thus we may say that within the great year's 12-brood data, there is no indication of chick parents playing favorites between the core and thus the marginal chick. Note that the identical way of calculations can also be applied for every brood game, especially the smaller ones (with but three or four players in one brood).

Here we provide another example within the great year's data using the above Shapley value calculation procedure to see if parental favoritism could exist even within the great years. Consider the 21-brood game under the restricted approach where there exist two core chick as player 1 and a pair of and one marginal as player 5.

Following the same Shapley value calculation procedure, ordering the chick in 21-brood into order 1-5-2 means that we firstly give allocation for coalition {1,3} (by giving allocation for chick 1 first from v({1,5}) and the rest for chick 5), then lastly give the rest of the grand coalition value v({1,2,5} for chick 2 after being reduced by v({1,5}). Table 10 below will list all possible orders additionally because the Shapley value for the 21-brood game mentioned earlier (see Table 3 for all the possible coalition values of this 21-brood game). In the end, we also compare the worth we got with our observation value for the 21-brood data (see the corresponding average of the survival rate for every chick involves within the 21-brood game from the A data).

Note that the sum of the observed average of all chick' survival rate within the corresponding game (see **d8 val** data in Table 2) is capable the worth of the grand coalition.

Table 10: Possible orders for 21-brood game, good years.

Possible Orders P	Player			
	1	**2**	**5**	**Total**
1-2-5	1.000	0.867	0.598	
1-5-2	1.000	0.603	0.862	
2-1-5	0.867	1.000	0.598	
2-5-1	0.603	1.000	0.862	
5-1-2	0.931	0.603	0.931	
5-2-1	0.603	0.931	0.931	
∅	0.834	0.834	0.797	2.465
Observ.	0.859	0.859	0.747	2.465

Source: Authors, (2019).

From Table 10, we see that the marginal chick gets a bit but its Shapley value solution, while the cores get a bit more within the observation. It implies that in keeping with the Shapley value solution concept and by considering every possible orders of feeding the chick within the good years 21-brood, we may say that the fogeys are quite 'favoriting' the core ones. Now we are going to jump over the unfeasible orders and consider only the feasible ones. Coalitions during which any marginal is fed before any core don't seem to be feasible. Thus, erasing orders 1-5-2, 2-1-5, 5-1-2, and 5-2-1 from our calculations will give us the table below:

Table 11: Feasible orders for 21-brood game, good years.

Feasible Orders P	Player			
	1	**2**	**5**	**Total**
1-2-5	1.000	0.867	0.598	
2-1-5	0.867	1.000	0.598	
∅	0.9335	0.9335	0.598	2.465
Observ.	0.859	0.859	0.747	2.465

Source: Authors, (2019).

We can see in Table 11 that if we remove the unfeasible coalition orders, the result's the opposite way around. Here the marginal gets way more in point of fact instead of what it speculated to get supported the Shapley value solution that we calculate.

Therefore, we'd like to test these two conditions on our Shapley value solutions: Whether the one claiming more will always get quite the one claiming less, and whether the one claiming more will always lose quite the one claiming less.

IV. RESULTS AND DISCUSSIONS

For the case of fine years 12-brood game, from Table 2 we will see that solely, players 1, 5, and 6 are claiming 1.000, 0.931, and 0.838, respectively. this suggests player 1 claims the foremost, while player 6 claims the smallest amount. When considering all possible orders P, the Shapley value solutions are 0.960, 0.879, and 0.786, respectively. Since in keeping with these solutions player 1 gets the foremost while player 6 gets the smallest amount, the primary property of the CG-solution is satisfied. However, once we consider the lose (i.e. the difference between the claim and also the reward) that each player has, player 1 loses 0.040, while players 5 and 6 equally lose 0.052. The loss of player 1 who claims the foremost, is of course smaller than the loss of two other players who claim less. Thus, the second property is unfortunately not satisfied. Therefore, when considering all the possible orders into the calculation, this Shapley value solutions of the great years 12-brood game isn't a CG-solution. we will also easily check for the case of removing the unfeasible orders and should attain the identical conclusion. within the case of the great years 21-brood game, we also get the identical conclusions when considering only the feasible orders into the Shapley value calculation.However, we get a special result once we consider all the possible orders. Claiming 1.000, 1.000, and 0.931 respectively, player 1 and player 2 equally get 0.834, while player 5 gets 0.797 in their Shapley value solutions. Claiming the foremost, players 1 and a pair of lose 0.166, while player 5 loses 0.134. we are able to easily see that this point, the 2 properties are satisfied. Thus, we may say that the Shapley value solutions of the great years 21-brood game are a CG-solution once we consider all the possible feeding orders into the calculation.

V. CONCLUSIONS

This paper gives a mild and gentle intro to cooperative game theory in chick's brood food allocation since the modeling and calculation choices we made using the Shapley value solution concept have proved that parental favoritism does exist in most cases of the brood data. The restricted structure of the brood datasets also enables the Shapley value solution concept to give a reasonable fit within a reasonable time. The brood data we are taking, the results of the experiments have tendency that small increase on the food allocation for the marginals could increase the chick's probability of survive a lot more, while giving more food to the core chick who already has a high survival rate does not give a different output as the core already has a great chance of surviving. To summarize the results of the experiments, the better the game fits the solutions, the more we can trust the resulting system. Shapley value solution concept to tackle the chick brood food allocation problem. We also successfully translate the biological problem of chick's food allocation into a cooperative game approach using various techniques known in literature.

VI. AUTHOR'S CONTRIBUTION

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