



DYNAMIC VIBRATION SUPPRESSION OF SMART COMPOSITE PLATES VIA LQR AND FINITE ELEMENT METHOD

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ABSTRACT

In this paper, a comprehensive finite element (FE) formulation is developed for the dynamic analysis and active control of classical laminated composite plates with embedded piezoelectric layers. The model, based on the First-Order Shear Deformation Theory (FSDT) and Hamilton's principle, accurately captures transverse shear effects and electromechanical coupling. Stiffness, mass, and piezoelectric contributions of actuator and sensor layers are explicitly included, providing a fully consistent multi-field representation of the intelligent structure. To enhance vibration performance, piezoelectric actuators are employed within an active vibration control (AVC) framework. A Linear Quadratic Regulator (LQR) controller is designed using the independent mode space approach, enabling efficient computation of optimal feedback gains for dominant vibration modes while keeping the control model reduced-order and computationally tractable. Numerical simulations on various laminated configurations show excellent agreement with ANSYS benchmark results in terms of natural frequencies and mode shapes. The LQR-based AVC strategy is shown to significantly reduce vibration amplitudes, with peak responses decreased by up to 92% within a few seconds and the damping ratio increased more than fivefold. Frequency-response analysis reveals up to 50% attenuation of resonance peaks, highlighting the synergistic effect of state-feedback control and piezoelectric coupling in increasing effective damping. These results confirm the effectiveness of the proposed FE-LQR approach in suppressing vibrations, accelerating structural stabilization, and enhancing the dynamic resilience of smart composite plates.



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I. INTRODUCTION

The increasing demand for lightweight, high-strength composite materials with integrated sensing and actuation capabilities has significantly expanded their applications in aerospace, automotive, and civil engineering [1], [2]. Laminated and sandwich composite plates embedded with piezoelectric layers have attracted substantial attention due to their ability to suppress vibrations and reduce noise [3], [4], [5]. However, the inherent anisotropy and high flexibility of such structures make them prone to excessive vibrations, necessitating the development of advanced control strategies to ensure structural stability [6], [7]. Active vibration control (AVC) techniques employing piezoelectric materials as sensors and actuators provide rapid response and efficient electromechanical coupling [8], [9].

Among these techniques, the Linear Quadratic Regulator (LQR) has emerged as a widely used method due to its ability to optimize control effort while maintaining system stability, effectively balancing vibration suppression with energy consumption [10], [11], [12]. Recent studies have demonstrated the effectiveness of LQR control in reducing peak vibration amplitudes by over 90% and enhancing damping ratios by several times compared to uncontrolled systems [13], [14]. A robust numerical modeling framework is crucial for designing an effective controller that accurately captures material anisotropy, layer interactions, and complex laminated geometries. The finite element method (FEM) provides a versatile tool for such purposes, allowing incorporation of both mechanical and electromechanical contributions of piezoelectric layers [15], [16], [17]. Recent works have extended FEM formulations to account for viscoelastic damping, functionally graded material (FGM) cores, and optimal placement of piezoelectric patches for enhanced vibration control [18], [19], [20]. Comparisons between FEM-based simulations and commercial software (e.g., ANSYS) have consistently confirmed the high accuracy of these models in predicting natural frequencies, mode shapes, and time responses of smart composite plates [15], [17], [18]. In this study, a finite element-based modeling framework is combined with an LQR-based active vibration control strategy for laminated composite plates embedded with piezoelectric patches. Numerical simulations demonstrate substantial reduction in vibration amplitudes, attenuation of resonance peaks, and accelerated structural stabilization. These results confirm the effectiveness of the proposed approach and contribute to the ongoing development of smart composite structures with enhanced dynamic performance and reliable active control capabilities.

II. FINITE ELEMENT FORMULATION

In this section, a finite element (FE) formulation is developed to model the mechanical and electromechanical behavior of laminated composite plates with embedded piezoelectric layers. The formulation combines the First-Order Shear Deformation Theory (FSDT) for accurate kinematics with piezoelectric constitutive equations, allowing the evaluation of mechanical displacements, strains, and electromechanical coupling. The FE discretization procedure is then presented to assemble the global system matrices for numerical analysis.

II.1 MECHANICAL DISPLACEMENTS AND STRAINS

The First-Order Shear Deformation Theory (FSDT) accounts for transverse shear deformation, which is ignored in Classical Plate Theory, to describe mechanical displacements and strains. By including the transverse shear effect, FSDT provides more accurate predictions for moderately thick laminated composite plates, compared to the classical theory which assumes that normals to the mid-plane remain perpendicular after deformation [21], [22]. This theory is widely used in the analysis of composite and smart structures due to its balance between accuracy and computational efficiency [23]–[25].

II.2 PIEZOELECTRIC CONSTITUTIVE EQUATIONS

The piezoelectric effect can be expressed by four pairs of equations linking the mechanical tensors of deformation $\{\epsilon\}$ and stress $\{\sigma\}$ to the vectors of the electric field $\{E\}$ and the induction (the electric displacement) $\{D\}$. It is common to present these equations in the following form[26]–[29]:

$$\begin{cases} \{\sigma\} = [Q]\{\epsilon\} - [e]^T\{E\} \\ \{D\} = [e]\{\epsilon\} - [\epsilon]\{E\} \end{cases} \quad (1)$$

$[Q]$ and $[e]$ are respectively called the elastic constants measured with constant electric field $\{E\}$, the piezoelectric constants and the measured dielectric constants with constant strain.

II.3 FINITE ELEMENT DISCRETIZATION

The shapes and structures of these matrices can be expressed in a standard coupled form, which is widely adopted for finite element modeling of piezoelectric laminated plates and smart structures [30]:

Matrix of masses

$$\begin{aligned} [M_e] &= \int [N]^T [\bar{m}] [N] dv = [M_{ep}, M_{ap}, M_{sp}] \\ &= \int_{-1}^1 \int_{-1}^1 [N]^T [\bar{m}] [N] |J| d\xi d\eta \\ &= \sum_{i=1}^{n_g} \sum_{j=1}^{n_g} [N]^T ([\bar{m}_p, \bar{m}_{ap}, \bar{m}_{sp}]) [N] |J| d\xi d\eta \\ [\bar{m}] &= \sum_{k=1}^n \int_{z_k}^{z_{k+1}} \begin{bmatrix} I_1 & 0 & 0 & I_2 & 0 \\ 0 & I_1 & 0 & 0 & I_2 \\ 0 & 0 & I_1 & 0 & 0 \\ I_2 & 0 & 0 & I_3 & 0 \\ 0 & I_2 & 0 & 0 & I_3 \end{bmatrix} dz, (I_1, I_2, I_3) \\ &= \sum_{k=1}^n \int_{z_{k+1}}^{z_k} \rho(1, z, z^2) dz \end{aligned} \quad (2)$$

Rigidity matrix

$$\begin{aligned} [K] &= \int_v [B]^T [\bar{D}] [B] dv = [K_{ep}, K_{ap}, K_{sp}] \\ &= \int_{-1}^1 \int_{-1}^1 [B]^T [D] [B] |J| d\xi d\eta \\ &= \sum_{i=1}^{n_g} \sum_{j=1}^{n_g} [B]^T ([\bar{D}_p, \bar{D}_{ap}, \bar{D}_{sp}]) [B] |J| d\xi d\eta \end{aligned} \quad (3)$$

Damping Matrix

$$[C] = \alpha[M] + \beta[K] \quad (04)$$

Electrical stiffness matrix

$$\begin{aligned} [K_{me}] &= \int_A [B]^T [e] [B_\phi] dA \\ &= \int_{-1}^1 \int_{-1}^1 [B]^T [e] [B_\phi] \\ &= \sum_{i=1}^{n_g} \sum_{j=1}^{n_g} [B]^T [e] [B_\phi] |J| d\xi d\eta \end{aligned} \quad (05)$$

Coupling matrix

$$\begin{aligned} [K_{ea}, K_{es}] &= \int_A [B_\phi]^T [\epsilon_{a,s}] [B_\phi] \\ &= \int_{-1}^1 \int_{-1}^1 [B]^T [e_{a,s}] [B_\phi] |J| d\xi d\eta \\ &= \sum_{i=1}^{n_g} \sum_{j=1}^{n_g} [B]^T [e] [B_\phi] |J| d\xi d\eta \end{aligned} \quad (06)$$

Mechanical forces

$$\begin{aligned} [F_m] &= \int_A [N]^T [d] dA = \int_{-1}^1 \int_{-1}^1 [N]^T [d] |J| d\xi d\eta \\ &= \sum_{i=1}^{n_g} \sum_{j=1}^{n_g} [N]^T [d] |J| d\xi d\eta \end{aligned} \quad (07)$$

III. ACTIVE VIBRATION CONTROL (AVC)

Active vibration control aims to suppress the dynamic response of flexible structures by using real-time feedback from sensors and applying appropriate control forces through actuators [31], [32]. In smart composite plates with integrated piezoelectric layers, this process requires an accurate state-space representation of the system to design an efficient controller such as the Linear Quadratic Regulator (LQR) [33], [34]. In this framework, the matrices [A], [B], and [C] correspond respectively to the system matrix, the control input matrix, and the output matrix. These matrices describe the structural dynamics, the influence of the actuator inputs, and the measured outputs used for feedback. They are derived by assembling the finite element mass, stiffness, and electromechanical coupling matrices into the standard state-space form [35], [36]:

The system matrix

$$A = \begin{bmatrix} 0 & I \\ -\text{diag}(\omega_k^2) & -2\xi_k \omega_k \end{bmatrix} \quad (08)$$

The control input matrix

$$B = \begin{bmatrix} 0 \\ \mu \Omega^T K^{(ac)}_{me} \end{bmatrix} \quad (09)$$

The output matrix

$$C = [K^{(l)}_{me} \quad \Omega \quad 0] \quad (10)$$

This state-space formulation provides the basis for implementing the LQR controller and achieving effective vibration suppression in intelligent composite structures.

III.1 OBJECTIVE FUNCTION

To design the Linear Quadratic Regulator (LQR) compensator, the control problem is formulated as the minimization of a quadratic performance index [37],[38]. This objective function represents a trade-off between the vibration energy of the structure and the control effort required by the actuators. The quadratic cost function is classically defined as:

The quadratic cost function

$$J = \int_0^\infty (\{X\}^T [Q] \{X\} + \{\varphi\}^T [R] \{\varphi\}) dt = \min \quad (11)$$

Where: Q is a positive semidefinite matrix and R is a positive matrix.

The optimal solution is:

The optimal solution

$$[G] = [R]^T [B]^T [K] \quad (12)$$

Where [K] satisfies the Riccati equation:

The Riccati equation

$$[A]^T [K] + [K] [A] - [K] [B] [R]^{-1} [B]^T [K] + [Q] = 0 \quad (13)$$

III.2 LOAD LINEAR QUADRATIC REGULATOR (LQR) PROBLEM

The Linear Quadratic Regulator (LQR) is a classical optimal control method used to compute a state-feedback control law that minimizes a quadratic cost function while ensuring system stability. In MATLAB, the command `lqr` is used to calculate the optimal gain matrix G .

The optimal gain matrix G

$$\text{Syntax : } [G, K, e] = \text{Lqr}(A, B, Q, R) \tag{14}$$

Where e is the closed-loop eigenvalues.

IV. NUMERICAL ANALYSIS AND DISCUSSIONS

In this section, some numerical results are presented for symmetrically laminated square plate $[45, -45, 45, -45]$, with embedded four identical piezoelectric actuators. The geometrical size of the intelligent composite plate is $L*L*H=1*1*0.002(m)$, for the piezoelectric patches are $l_p*l_p*h_p=0.1*0.1*0.0001(m)$ (Fig.1). The localizations for the piezoelectric actuators are $(X1, Y1) = (0.2, 0.4)$, $(X2, Y2) = (0.4, 0.6)$, $(X3, Y3) = (0.8, 0.7)$, $(X4, Y4) = (0.4, 0.9)$ (Figure.1).

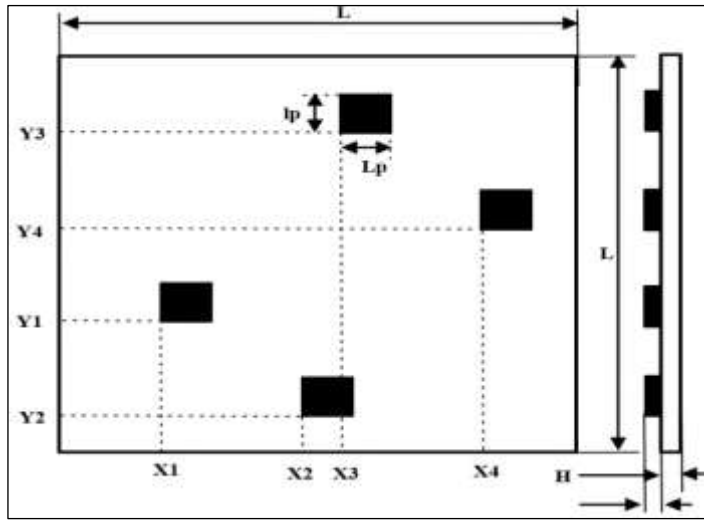


Figure 1: The geometrical model of the plate equipped with PZT patches.
Source: Authors, (2026).

The simulation featured in this paper assume $\alpha=0$ and $\beta=0.0015$ damping constants. The time step Δt for Transient analysis is taken as $1 / (20f_h)$, where f_h is the higher frequency. Consider an initial displacement field applied to the plate equal to 0.1 m. Table 1 contains the material properties data for the plate, piezoelectric actuators.

Table 1: Material properties.

properties	Graphite/epoxy	PZT G-1195
Poisson's ratio	0.31	0.3
Density ρ (kg/m ³)	1550	7600
Elastic stiffness matrix (GPa)		
E11	119	132.38
E22	8.67	10.76
E33	8.67	10.76
G12	5.18	3.61
G13	3.29	5.61
G23	3.29	5.61
Piezoelectric constant (C/m ²)		
e31	-	12.5
e33	-	12.5
e15	-	-
Dielectric constant (F/m)		
g11	-	1.53×10^{-8}
g22	-	1.53×10^{-8}
g33	-	1.53×10^{-8}

Source: Authors, (2025).

A finite element (FE) model, created using Matlab, facilitates the dynamic modeling of a system. Numerical results are verified using ANSYS, where an FE model of an intelligent composite plate is developed. The solid layers and PZT materials are modeled with SOLID45 and SOLID5 elements, featuring 8 nodes. A coupling electromechanical system is established using the CP command, and a suitable voltage potential is assigned (Figure 2).

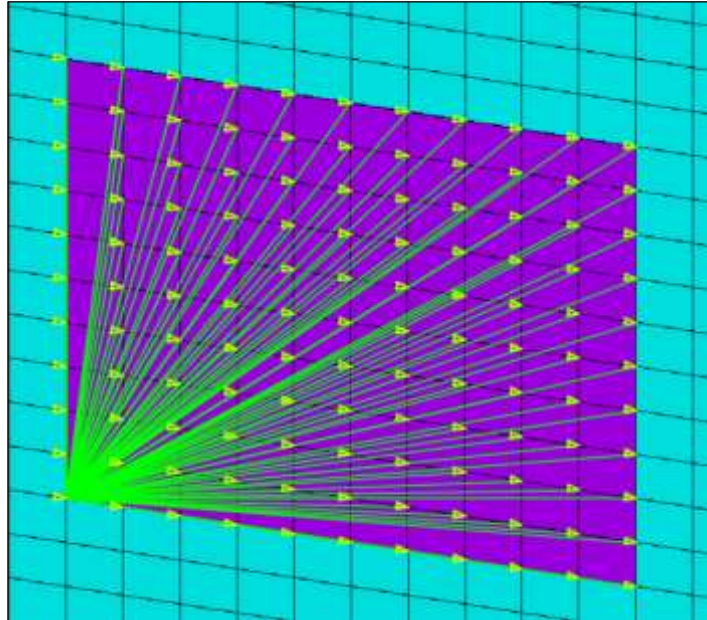


Figure 2: A coupling electromechanically created by CP command.
Source: Authors, (2026).

The current cantilever intelligent composite plate is meshed by global element size $e_{big}=10e^{-3}$, eight-node solid elements (Figure 3).

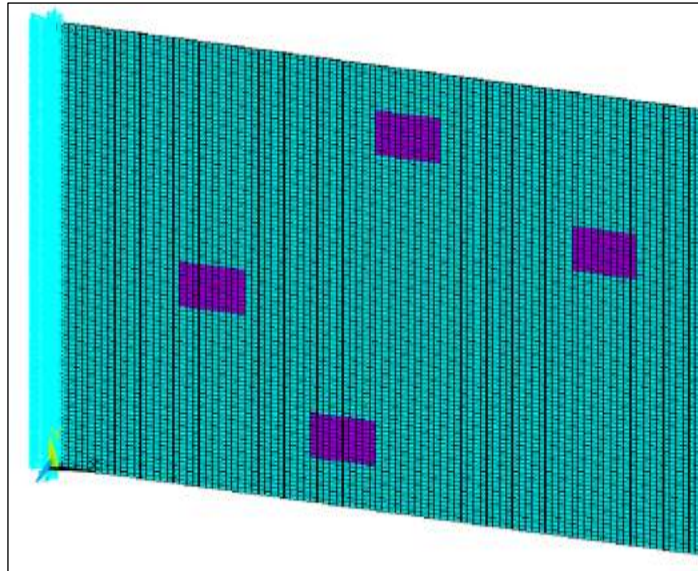


Figure 3: FEM of the cantilever intelligent composite plate and boundary conditions.
Source: Authors, (2026).

The modal analysis of the system reveals that the first three natural frequencies (Table 2) of the system lie in the range 110Hz.

Table 2: The first three natural frequencies (Hz) of the intelligent plate.

	Matlab	Ansys	Error (%)
First Mode	2.3219	2.5297	3.14
Second Mode	5.0515	5.7154	5.24
Third Mode	7.2065	7.3853	1.49

Source: Authors, (2026).

In this study, a linear quadratic optimal controller is considered to control the first three modes of the flexible plate. The weighting matrices chosen are: $Q=5e^{-8}$ and $R=0.5e^{+10}$. Here, the control is started after an elapse of 0.5 s in order to compare the controlled and uncontrolled responses. Figure 04 shows the displacement responses of the uncontrolled and controlled intelligent composite plate.

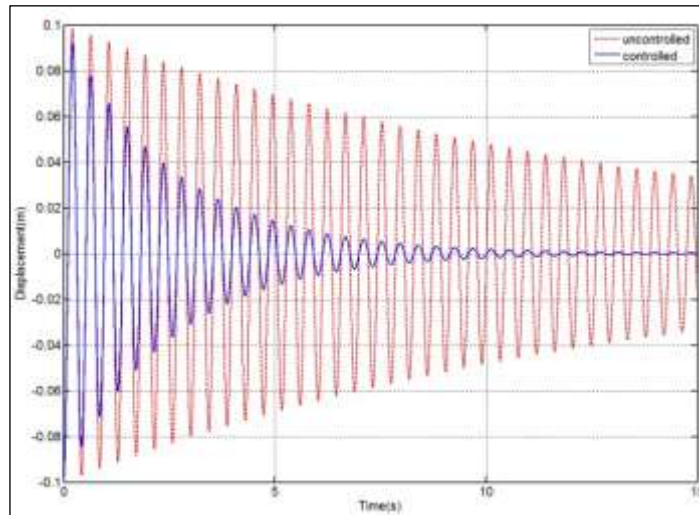


Figure 04: Comparison of uncontrolled and controlled displacement response of an intelligent composite plate with LQR controller.
Source: Authors, (2026).

The quantitative analysis of the responses shows that the introduction of LQR active control significantly improves the dynamic behavior of the composite plate. The maximum amplitude is reduced by approximately 36% from the initial instant, and by nearly 92% after 5 seconds, compared to the uncontrolled case. The calculation of the logarithmic decrement indicates that the damping ratio increases from about 0.61% (uncontrolled structure) to 3.3% under control, corresponding to an increase by a factor of ≈ 5.4 . Moreover, the controlled system reaches a quasi-steady state in less than 4 seconds, whereas the uncontrolled system continues to vibrate with high amplitudes throughout the simulation window. These results clearly demonstrate the effectiveness of the LQR controller in enhancing effective damping, reducing vibrational energy, and accelerating the stabilization of the structure. The frequency response graph in Figure 5 compares the uncontrolled and controlled vibration amplitudes of the intelligent composite structure over the frequency range of 10–100 rad/s.

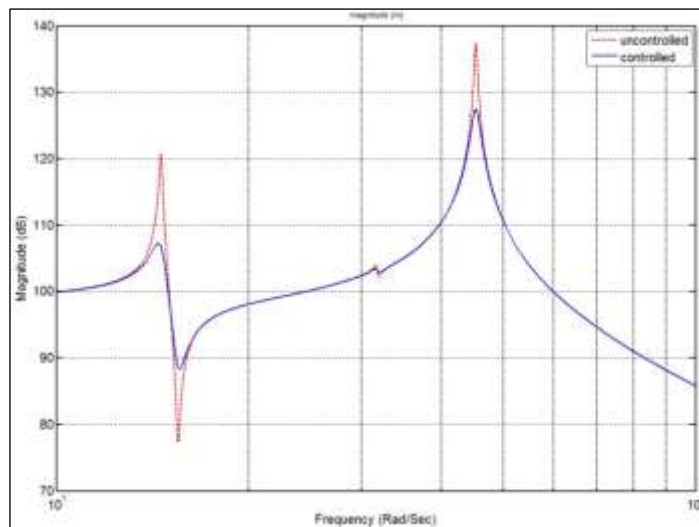


Figure 5: Comparison of the uncontrolled and controlled frequency response of an intelligent composite plate with LQR controller.
Source: Authors, (2026).

The controlled frequency-response curve exhibits a marked reduction in resonance amplification, with the main peak decreasing from approximately 133 dB (uncontrolled) to about 125 dB (controlled), corresponding to nearly a 6–8 dB attenuation which represents a reduction of about 40–50% in vibration energy; this flattening of the frequency response is consistent with the expected behavior of optimal state-feedback controllers, and is further enhanced by the piezoelectric coupling mechanism through which part of the induced strain energy is transformed into electrical energy, thereby increasing the effective damping of the structure.

V. CONCLUSIONS

In conclusion, the results demonstrate that the combined finite element modeling and LQR control strategy effectively reduces vibrations in smart composite plates with piezoelectric layers. Comparison of simulations in MATLAB and ANSYS confirms the accuracy of the model, as the natural frequencies match closely with minimal error. The numerical analyses further show that the LQR controller significantly enhances the dynamic performance of the plate, reducing peak vibration amplitudes by up to 92% within a few seconds and increasing the damping ratio more than fivefold. Frequency-response analysis highlights a substantial attenuation of resonance peaks, with up to 50% reduction in vibration energy, reflecting the synergistic effect of state-feedback control and piezoelectric coupling in increasing effective damping. Overall, these findings validate the LQR controller's effectiveness in suppressing vibrations, accelerating structural stabilization, and improving the dynamic resilience of smart composite structures.

VI. AUTHOR'S CONTRIBUTION

Conceptualization: Mourad Chalane*, Mohamed Latrache and Bachir Labiodh.

Methodology: Mourad Chalane*, Mohamed Latrache and Bachir Labiodh.

Investigation: Mourad Chalane*, Mohamed Latrache and Bachir Labiodh.

Discussion: Mourad Chalane*, Mohamed Latrache and Bachir Labiodh.

Writing: Mourad Chalane*, Mohamed Latrache and Bachir Labiodh.

Writing: Mourad Chalane*, Mohamed Latrache and Bachir Labiodh.

Resources: Mourad Chalane*, Mohamed Latrache and Bachir Labiodh.

Supervision: Mourad Chalane*, Mohamed Latrache and Bachir Labiodh.

Approval of the final text: Mourad Chalane*, Mohamed Latrache and Bachir Labiodh.

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