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RESEARCH ARTICLEOPEN ACCESS

ANALYTICAL AND NUMERICAL MODELING OF THE TRANSIENT BEHAVIOR OF AN EARTH CONNECTION DURING THE INJECTION OF AN ELECTROMAGNETIC WAVE

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ABSTRACT

This paper is devoted to the study and modeling of the behavior of a grounding system. The latter is used for the protection of electrical installations and equipment against surges and various disturbances affecting these systems. The methods used for modeling are multiple for this, an analysis and a synthesis of these methods has been made. In this work, the Agrawal model was used, which is based on the line theory. The finite integration technique was used under the CST Software for the purpose of verifying the first results. At the end, the different factors that influence the response of grounding systems were studied to evaluate the impulse performance. First, we start by determining the current distribution along the excited electrode in order to characterize the grounding radiation over time. The last part of this work is based on a parametric study that takes into account the electrode burial depth as well as the resistivity jump between the electrode and the surface. The results obtained by our analytical model are compared and validated by the CST/EMC software. Good agreements are found between these approaches.

I. INTRODUCTION

Many Several problems in the electrical engineering, electronics or telecommunications industries are due to short circuits or the direct impact of lightning waves on their networks, which put them in a disturbed state, for this reason the analysis of the behavior of grounding in disturbed conditions remains among the main concerns. Short circuits or the direct impact of lightning waves on networks can put certain metal parts equipping a THT or TH station under voltage and for the protection of these devices is the earth network which must limit the duration of this voltage and allow a rapid flow of charges into the ground [1-3].

This discharge into the ground of an overhead station has the following effects: to bring the masses of the latter to a certain potential which risks causing return sparks between the masses and other circuits entering the station to generate radiation likely to disturb the electromagnetic environment (EM) of local electrical systems (self-pollution), and to generate a coupling with neighboring lines [4].

The measurement of earthing is still affected by problems that often lead to considerable uncertainty when interpreting the measured values [5-8].This leads researchers to approach this problem by mathematical calculation using some simplifying assumptions [9]. In this work, we are interested in the transient modeling of a buried electrode, and then we will study the radiation and the coupling of this one with an aerial structure [10-12].

II. LINE THEORY

In It is known that it is necessary to build the line theory on the properties of the electromagnetic field that is to say on Maxwell's equations. We seek solutions in the form of a wave propagating parallel to the line and compatible with the boundary conditions, then we try to find an "equivalent scheme" which gives the same solutions from Ohm's law, which is more convenient to use than Maxwell's equations. Due to the wire geometry of the ground electrodes, it is possible to study them using line theory [1]. The advantage of this theory is that it facilitates the determination of input impedance at the end of the antenna (Figure 1). This is characterized by its impedance per unit length Z and its admittance per unit length Y.

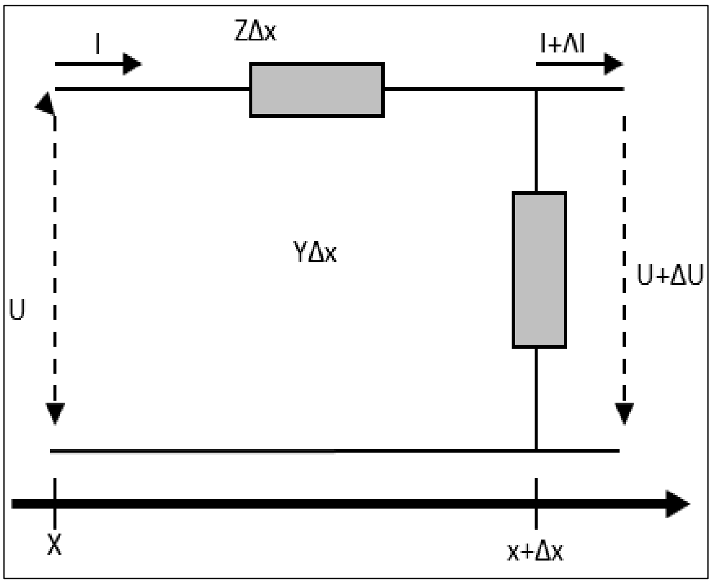

Figure 1: Equivalent schema of a line section.
Source: Authors (2025).

The assumption required for such modeling is that any propagation effect is negligible for each wire element of the structure. The main step is therefore the characterization of the elementary section of the line for which the circuit laws (node law, mesh law, Ohm's law, etc.) can be written locally [2].

For this, each section taken separately must be independent of its neighbors; consequently it is necessary to consider a wire structure of infinite length located in an infinite medium. The introduction of a finite length will be carried out a posteriori as a discontinuity at each end. Since the transverse dimensions of the line are small compared to the wavelength, this discontinuity can be taken into account by writing a local condition [3].

The line equivalent to a straight wire structure of infinite length, immersed in an infinite medium, is now well determined by its primary parameters Z and Y which are independent of the propagation and characteristic of the physical model, and by its secondary parameters k and Zc. The ground electrode is then considered as a transmission line with losses. In frequency for an infinitesimal element of the electrode, the current and voltage equations are given by [4]:

$$\begin{cases} \dfrac{dU}{dx} = -ZI \\ \dfrac{dI}{dx} = -YU \end{cases} \tag{1}$$

The following telegraphist equations:

$$\begin{cases} \dfrac{d^2U}{dx^2} = \gamma^2 U \\ \dfrac{d^2I}{dx^2} = \gamma^2 I \end{cases} \tag{2}$$

Are satisfied by two independent solutions that represent waves moving in positive and negative directions:

$$\begin{cases} U(x) = \left(Ae^{-\gamma x} + Be^{\gamma x}\right)Z_c \\ I(x) = \left(Ae^{-\gamma x} - Be^{\gamma x}\right) \end{cases} \tag{3}$$

A and B are constants of integration.

$$\begin{aligned} Z &= R + j\omega L \\ Y &= G + j\omega C \end{aligned} \tag{4}$$

With:

$Z_c = \sqrt{\dfrac{Z}{Y}}$: Characteristic impedance

$\gamma = \sqrt{ZY} = \alpha + j\beta$: Wave propagation coefficient

α : Linear attenuation

β : Linear phase delay

In this research work we will present a semi-analytical formalism to quantify the coupling of a buried electrode, subjected to excitation, with a piece of overhead line. This formalism is a combination of several methods: First, we start by determining the distribution of the current along the excited electrode and this by using the theory of transmission lines. To characterize the radiation of a grounding in time, we used the dipole method for the quantification of the electromagnetic field radiated by the latter in a free space and the modified image method to take into account the effect of the interface. In the last step, which consists of the study of the electromagnetic coupling, which manifests itself by induced currents and voltages, we will use the Agrawal model [9].

This model does not lack precision and lends itself better to numerical modeling. . Since the soil is relatively poorly understood (its properties change with temperature and humidity), the treatment of earthing by rigorous formalisms is only fortuitous.

III. COUPLING EQUATION FOR THE CASE OF AN IDEAL CONDUCTOR ABOVE A PERFECTLY CONDUCTING GROUND

It is known that the theory of lines is built on the properties of the electromagnetic field, i.e. Maxwell's equations. Under certain approximations and assumptions, it is then possible to translate equations linking the electric and magnetic fields into expressions directly showing the current and voltage. The basic assumptions of the approximation of transmission lines are [5].

- The geometry of the line is reasonably uniform;
- The transverse dimensions of the line are smaller than the minimum significant wavelength;
- The conservation of current (sum of the forward and reverse currents is equal to zero);

- The quasi-transverse electromagnetic (quasi-TEM) propagation mode is predominant along the line. Consider a transmission line formed by a conductor of radius a located at a height h above a perfectly conducting ground (Figure 2).
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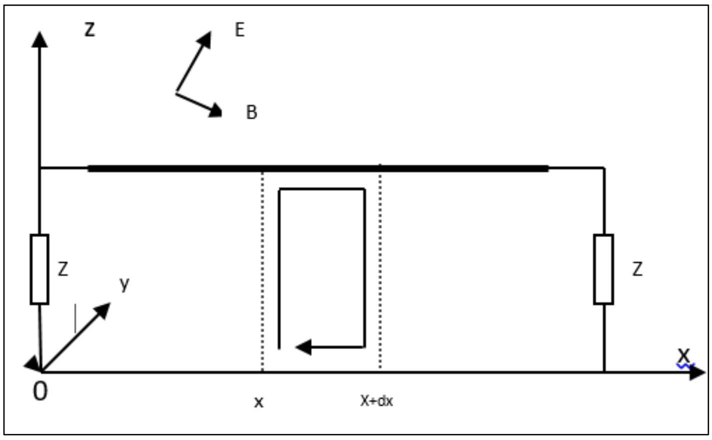

Figure 2: Geometry of a line formed by a conductor illuminated by an electromagnetic field.
Source: Authors (2025).

The exciting electromagnetic fields E^e and B^e are defined as the sum of the incident fields E^{inc}, B^{inc} and the fields reflected by the ground E^{ref}, B^{ref} in the absence of the conductor [7],[8].

$$\vec{E}^e = \vec{E}^{inc} + \vec{E}^{ref} \quad (5)$$

$$\vec{B}^e = \vec{B}^{inc} + \vec{B}^{ref} \quad (6)$$

The total fields E and B are obtained by adding the exciting fields E^e, B^e, and the diffracted fields, E^s, B^s which represent the reaction of the line to the exciting field.

$$\vec{E} = \vec{E}^e + \vec{E}^s \quad (7)$$

$$\vec{B} = \vec{B}^e + \vec{B}^s \quad (8)$$

IV. AGRAWAL MODEL

In this formulation, the two equations introduced by Agrawal et al [9]. deduced from equations (2) and (8), are expressed in terms of diffracted voltage and total currents:

$$\frac{dU^s(x,t)}{dx} + L\frac{\partial I(x,t)}{\partial t} = E_x^e(x,h,t) \quad (9)$$

$$\frac{dI(x,t)}{dx} + C\frac{\partial U^s(x,t)}{\partial t} = 0 \quad (10)$$

With :
$E_x^e(x,h,t)$ is the horizontal component of the exciting electric field along the axis at the height h of the conductor;
$U^s(x,t)$ is the vertical component of the diffracted magnetic field

$$U^s(x,t) = -\int_0^h E_z^s(x,z,t)\,dz \quad (11)$$

With:
$E_z^s(x,z,t)$ is the vertical component of the diffracted electric field.

The total voltage induced on the line $U(x,t)$ can be expressed as a function of the diffracted voltages $U^s(x,t)$ and the exciting voltages $U^e(x,t)$ by the following relation:

The boundary conditions for the diffracted voltage are:

$$U^s(0,t) = -Z_A I(0,t) + \int_0^h E_z^e(0,z,t)\,dz \quad (12)$$

$$U^s(L,t) = Z_B I(L,t) + \int_0^h E_z^e(L,z,t)\,dz \quad (13)$$

In the Agrawal model, the horizontal component of the exciting field along the line and the exciting vertical electric field at the vertical ends of the line are terms that appear explicitly in the equations to produce the diffracted voltage. Agrawal et al proposed an equivalent coupling circuit described by the two equations (7) and (8). This circuit is shown in Fgure 3

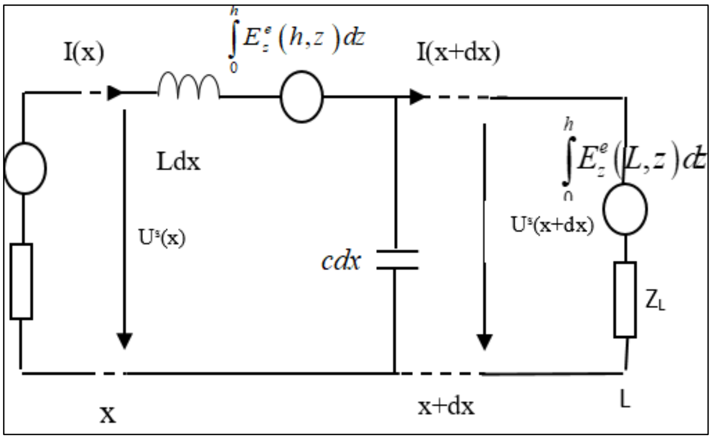

Figure 3: Equivalent differential coupling diagram according to the Agrawal et al model.
Source: [9].

Although the Agrawal model is the most suitable according to the literature [6], it is important to observe that an equivalent formulation of the transmission line coupling equations had been proposed by [10]:

$$\frac{dU(x,t)}{dx} + L\frac{\partial I(x,t)}{\partial t} = -\frac{\partial}{\partial t}\int_0^h B_y^e(x,z,t)\,dz \quad (14)$$

$$\frac{dI(x,t)}{dx} + C\frac{\partial U(x,t)}{\partial t} = -C\frac{\partial}{\partial t}\int_0^h E_z^e(x,z,t)\,dz \quad (15)$$

The boundary conditions for tension are:

$$U(0,t) = -Z_A I(0,t) \quad (16)$$

$$U(L,t) = Z_B I(L,t) \quad (17)$$

Agrawal's model is numerically more interesting than the others, because it involves only one source term in one of the two equations; and this source term does not contain any differentiation with respect to time and space.

IV.1. COUPLING EQUATION FOR THE CASE OF A LINE WITH FREQUENCY LOSSES

Consider the same configuration of Figure 2 but taking into account the losses in the conductor and in the ground. The conductivity of the conductor is σ_w and the ground is characterized by a conductivity σ_g and a relative permittivity ε_{rg} .

After correction due to the effect of the ground (finite conductivity) which consists in taking the latter as a dissipative half-medium, the frequency coupling equations in this case become [11].

$$\frac{dU^s(x)}{dx} + ZI(x) = E_x^e(x,h) \tag{18}$$

$$\frac{d\,I(x)}{dx} + YU^s(x) = 0 \tag{19}$$

With:

$$Z = j\omega L + Z_w + Z_g \tag{20}$$

$$Y = \frac{(G + j\omega C)Y_g}{G + j\omega C + Y_g} \tag{21}$$

$$Y_g = \frac{(G + j\omega C)\,j\omega L}{Z_g} \tag{22}$$

With : Z_w is the linear internal impedance of the conductor given:

$$Z_w = \frac{\gamma_w\, I_0(\gamma_w a)}{2\pi a\,\sigma_w I_1(\gamma_w a)} \tag{23}$$

$$\gamma_w = \sqrt{j\omega\mu_0}\left(\sigma_w + j\omega\varepsilon_0\varepsilon_{rw}\right) \tag{24}$$

With:

γ_w Propagation constant of the conductor medium, I_0 and I_1 are the modified Bessel functions.

Z_g Ground impedance given by a logarithmic approximation which was developed by sunde [12].

$$Z_g = \frac{j\omega\mu_0}{2\pi}\ln\left(\frac{1+\gamma_g h}{\gamma_g h}\right) \tag{25}$$

With: γ_g Propagation constant in the ground.

$$\gamma_g = \sqrt{j\omega\mu_0}\left(\sigma_g + j\omega\varepsilon_0\varepsilon_{rg}\right) \tag{26}$$

IV. 2. COUPLING EQUATION FOR A LINE IN THE TIME DOMAIN

The coupling equations in the time domain are obtained by the Fourier transformation of equations (18) and (19). The convolution product highlights the frequency dependence of the correction terms [13].

$$\frac{dU^s(x,t)}{dx} + L\frac{\partial I(x,t)}{\partial t} + \int_0^t Z(t-\tau)I(x,t)\,d\tau = E_x^e(x,h,t) \tag{27}$$

$$\frac{dI(x,t)}{dx} + \int_0^t y(\tau)U^s(x,t-\tau)\,d\tau = 0 \tag{28}$$

Where:

$Z(t)$ and $Y(t)$ are respectively the inverse Fourier transforms of $Z_g + Z_w$ and Y .

For practical cases, equation (28) can be approximated by:

$$\frac{d\,I(x)}{dx} + G\,U^s(x) + C\frac{\partial U^s(x)}{\partial t} = 0 \tag{29}$$

V. CST SOFTWARE

Generally speaking, the sequence of CST Software programs, equipped with an interface schematized by figure.4 [14-17].

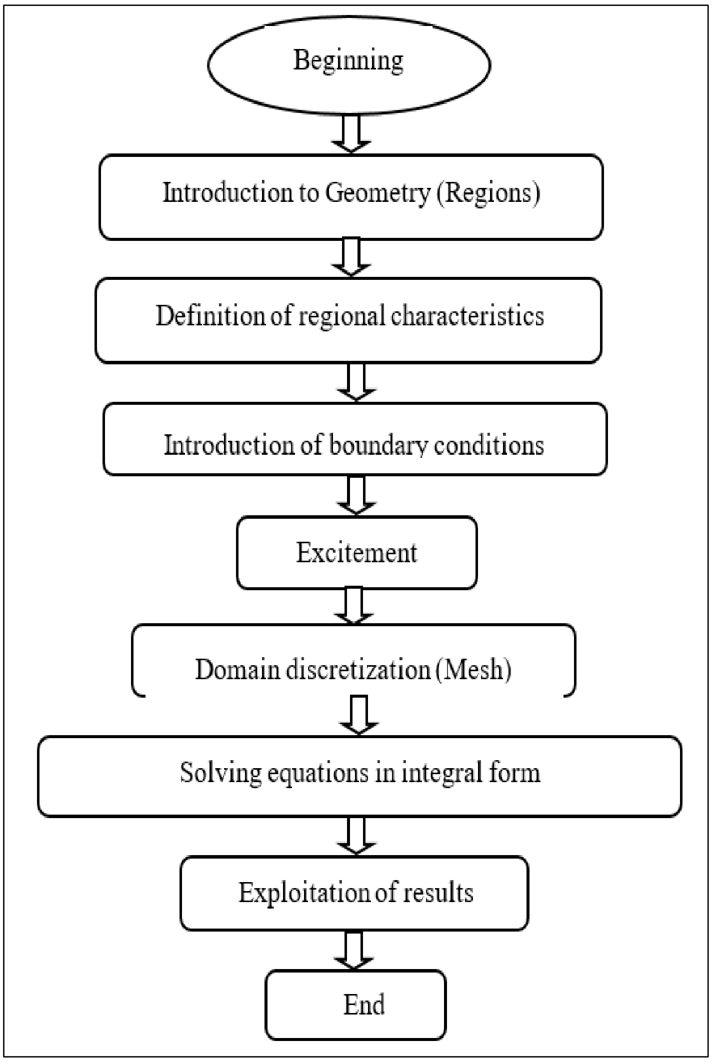

Figure 4: Flowchart of the different modules of the CST software.
Source: Authors (2025).

VI. RESULTS AND APPLICATIONS

VI.1. THE CURRENT AND THE POTENTIAL AT SEVERAL POINTS OF THE ELECTRODE

This is a rectilinear electrode of radius “a” and length “l” (figure 5), buried at a depth “h” in a soil of resistivity ρ, homogeneous and isotropic linear, and supplied at one of its extrinsics by a “lightning wave” generator.

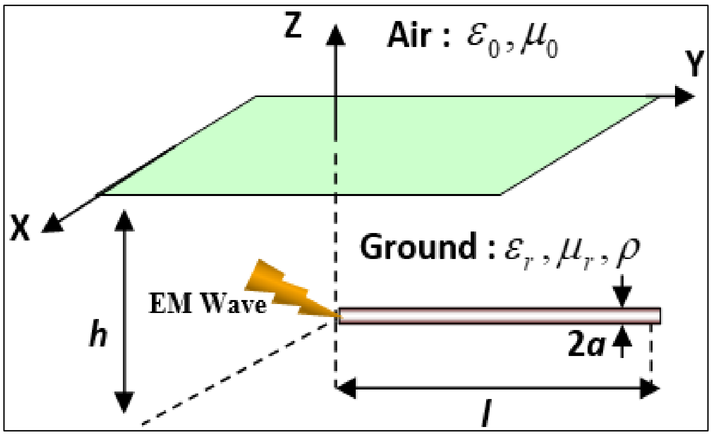

Figure 5: Electrode buried horizontally.
Source: Authors (2025).

For the validation of the computer implementation of the resolution of the equations of the Agrawal model, we consider a rectilinear electrode of radius 7 mm, of a length of 20 m, buried in a linear, homogeneous and isotropic ground at a depth of 0.8 m, the soil has resistivity ρ = 320 Ω.m, electrical permittivity εr = 36 and magnetic permeability μr = 1 as shown in figure 4. The electrode is attacked by a source of disturbance

- $V(t) = V_0(\exp(-\alpha t) - \exp(-\beta t))$
- $V_0 = 30\ KV, \alpha = 45099\mu s^{-1}, \beta = 9022879\mu s^{-1}$

To validate this first work, we compare the results obtained with the Agrawal model with those obtained using the CST Software which uses the rigorous antenna formalism. Let us recall that the antenna formalism consists in solving an integral equation in the electric field E or in the magnetic field H by the numerical finite element method.

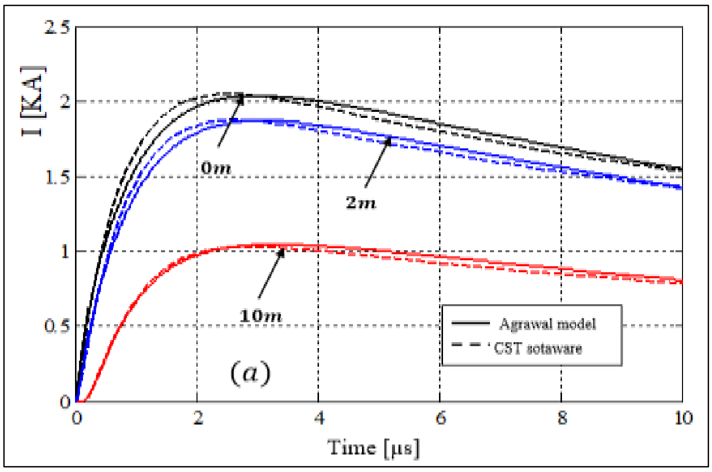

Figure 6: Variation of the current at several points of the electrode.
Source: Authors (2025)

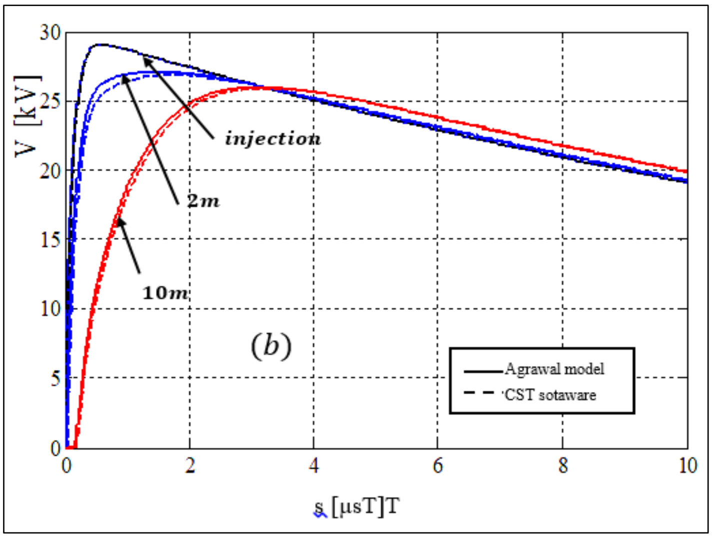

Figure 7: Variation of the potential at several points
Source: Authors (2025)

VI. 2. VARIATION OF THE ELECTRIC AND THE MAGNETIC FIELD

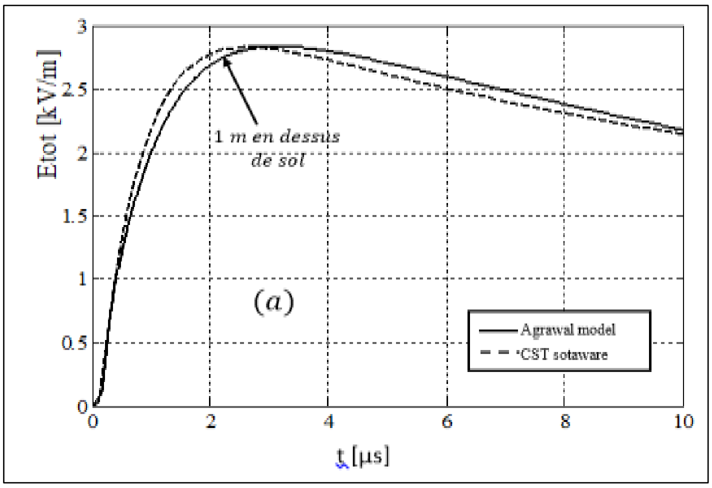

Figure 8: Variation of the electric field at 1m above the ground.
Source: Authors (2025).

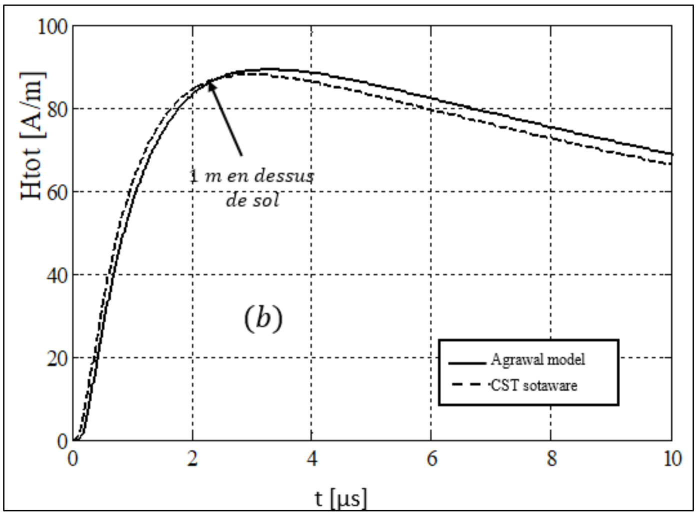

Figure 9: Variation of the magnetic field at 1m above the ground.
Source: Authors (2025).

Figures 6 to 9 show respectively the temporal variation of the current and voltage at several points of the electrode (0, 2 and

10 m) (Figure 6 and 7), as well as the temporal variation of the electromagnetic field at 1 m above the ground considered (Figure 8 and 9). These results in these figures clearly show that the results obtained by the Agrawal analytical model are very close to those obtained by the finite element method by the commercial CST software, the slight difference is mainly related to the use of time-frequency passages by fast Fourier transform when we use the CST Software software.

VI.3. PARAMETRIC STUDY

To consolidate our modeling which consists in associating the concept of apparent resistivity with the topological formalism, we propose a parametric study which takes into account the burial depth of the electrode as well as the resistivity jump between the two layers (ρ1>ρ2 and ρ1<ρ2).

VI.3.1. THE FIRST CASE

In the first case, the resistivity of the upper layer is fixed at ρ1=200 Ω.m and the resistivity of the lower layer successively takes the following values:

- $\rho_2 = 1000\ \Omega.m$
- $\rho_2 = 2000\Omega.m$
- $\rho_2 = 4000\Omega.m$

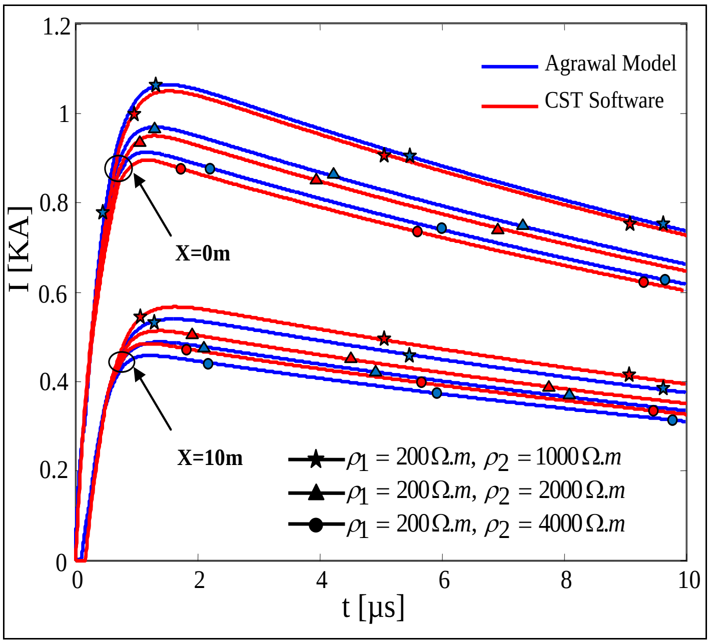

Figure 10: Temporal variation of the current at two points of the electrode (x=0m and x=10m).
Source: Authors (2025).

VI.3.2 THE SECOND CASE

In the second case, the value of the resistivity of the upper layer is fixed at ρ1=1000 Ω.m and the resistivity of the lower layer ρ2 successively takes the following values:

- $\rho_2 = 50\ \Omega.m$
- $\rho_2 = 100\Omega.m$
- $\rho_2 = 200\Omega.m$

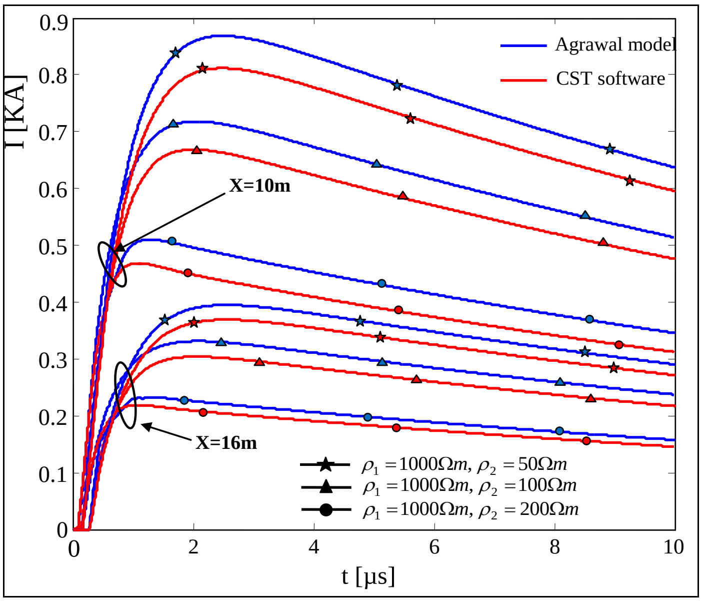

Figure 11: Temporal variation of the current at two points of the electrode (x=0m and x=10m).
Source: Authors (2025).

VI.3.3 THE THIRD CASE

For the third case, we consider a stratified soil with ρ1=200 Ω.m and ρ2=1000 Ω.m, but the burial depth of the electrode successively takes the following values:

- $h = 0.4\ m$
- $h = 0.8\ m$
- $h = 1.2\ m$

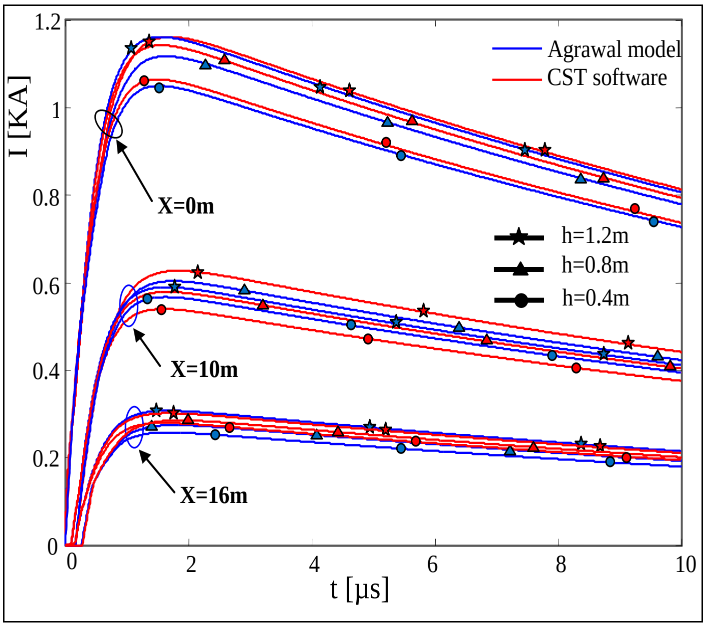

Figure 12: Temporal variation of the current at different points of the electrode.
Source: Authors (2025).

The temporal variations of the currents in Figures 10 to 12 clearly show that the resolution of the Agrawal equations for a two-layer laminated soil in the presence of an electrode as well as the CST Software simulation software lead to comparable results in amplitude and shape. These results confirm that the concept of apparent resistivity is not affected by the jumps in conductivity or by the depth of burial of the electrode.

VII. CONCLUSION

Our research work focuses on the calculation of the transient electromagnetic field emitted by a buried electrode (electromagnetic pollution) and the induced currents and voltages generated in an overhead line that lead to electromagnetic coupling. In this work we propose the use of a semi-analytical formalism allowing the quantification of the coupling between a horizontally buried electrode and excited by a current in the form of a lightning wave with an overhead wire structure, directly in time.

This quantification was done in several steps. The first step consists in determining the distribution of the current and the potentiel along the buried electrode, for this we used the modeling by the theory of transmission lines. The second step consists in the quantification of the electric and magnetic field radiated by the excited electrode. The last step consists in determining the distribution of the current and the potential induced by electromagnetic coupling, along the piece of line and this by using the Agrawal coupling model and CST Software.

The comparison of the results obtained by this analytical formalism (Agrawal model) with those obtained by the CST Software software, showed a fairly good agreement both in shape and amplitude. We can therefore say that this model constitutes a fast and fairly simple way to evaluate the electromagnetic coupling between two structures.

V. AUTHOR'S CONTRIBUTION

Conceptualization: Mohammed chebout, Hakim Azizi, Daoud Sekki, Mohammed Charif Kihal and Marouane Kihal.

Methodology: Mohammed chebout, Hakim Azizi and Daoud Sekki.

Investigation: Hakim Azizi, Mohammed chebout and Mohammed Charif Kihal.

Discussion of results: Mohammed chebout, Hakim Azizi and Daoud Sekki.

Writing – Original Draft: Mohammed chebout, Hakim Azizi, Mohammed Charif Kihal and Marouane Kihal**.**

Writing – Review and Editing: Hakim Azizi, Mohammed Charif Kihal and Marouane Kihal**.**

Resources: Mohammed chebout, Hakim Azizi, Daoud Sekki, Mohammed Charif Kihal.

Supervision: Hakim Azizi, Mohammed Charif Kihal and Marouane Kihal**.**

Approval of the final text: Mohammed chebout, Hakim Azizi, Mohammed Charif Kihal and and Daoud Sekki.

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